MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

EMT1026 – ENGINEERING MATHEMATICS II

(All Sections / Groups)

1 MARCH 2016

9.00 AM – 11.00 AM

(2 Hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **9 pages** (including the cover page and appendices) with **4** questions only.

2. Attempt **all** 4 questions. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

Question 1

(a) Find the general solution of the following differential equation:

$$y'' - 8y' + 12y = e^{3x} + 4$$

[12 marks]

(b) Using the power series method, find the first six non-zero terms of the solution for the following differential equation:

$$y'' - y' = 0$$

[13 marks]

Question 2

(a) Suppose the temperature distribution, $u(x,t)$, in a uniform bar of length 4 is modeled by the following partial differential equation:

$$u_t = 10u_{xx}, \text{ for } 0 \le x \le 4,\ t > 0,$$

with boundary conditions $u(0,t) = u(4,t) = 0$. Using the method of separation variables, show that the general solution of the above boundary value problem is

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{4}x\right) e^{-\left(\frac{n\pi}{4}\right)^2 10t},$$

where b_n are constants. Show all the relevant steps leading up to the solution.

[18 marks]

(b) From Question 2(a), if the initial condition is given as $u(x,0) = 20, 0 < x < 4$, then find the temperature distribution of the bar. (The formulae in APPENDIX A may be useful for solving this question.)

[7 marks]

Continued...

Question 3

(**Note**: The tables in APPENDIX B and APPENDIX C may be useful for solving this question.)

(a) Let $g(t)=\frac{t}{3}e^{2\omega t}$, where ω is a constant. Find the Laplace transform of $g(t)$

(i) using the s-Shifting property. [3 marks]

(ii) using the Derivative of Transform property. [3 marks]

(b) Let $G(s)=\frac{2}{s(s^2+1)}$. Find the inverse Laplace transform of $G(s)$

(i) using the Transform of Integral property. [3 marks]

(ii) using the Convolution Theorem. [3 marks]

(c) By using the method of Laplace transforms, solve the following differential equation:

$$y''-y'-2y=e^{2t},\ y(0)=0,\quad y'(0)=1.$$

[13 marks]

Continued...

Question 4

(**Note**: The tables in APPENDIX D, APPENDIX E and APPENDIX F may be useful for solving this question.)

(a) A certain brand of car tyre can be safely used over an average distance of $\mu = 25000$ km. The distance covered before a tyre needs changing is normally distributed with a standard deviation of $\sigma = 2000$ km.

(i) What is the probability that a tyre lasts for between 23500 km and 26500 km?

[4 marks]

(ii) Each tyre comes with a warranty for 22000 km. Out of every 1000 tyres, how many would probably qualify for a warranty claim?

[4 marks]

(b) Based on historical statistics, it is determined that the soccer team ManYou scores an average of 2.4 goals per soccer game. A typical soccer games lasts 60 minutes. Suppose ManYou maintains its performance, find the probability that the team scores

(i) more than 2 goals in an upcoming game.

[4 marks]

(ii) 1 goal in the first-half of the game and another goal in the second-half.

[3 marks]

(c) A continuous random variable X has probability density function

$$f(x) = \begin{cases} k\left(4 - x^2\right), & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Show that $k = 3/16$, then answer any TWO of the following:

(i) Find the mean of X, hence evaluate $E[8X + 8]$.

(ii) Find the variance of X.

(iii) Prove that if $P(X \le m) = \frac{1}{2}$, then $m^3 - 12m + 8 = 0$.

[10 marks]

End of questions.

APPENDIX A:

Half-range sine series:

$$f(x)=\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right), \quad \text{where } b_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

Half-range cosine series:

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right), \quad \text{where } a_0=\frac{2}{L}\int_0^L f(x)dx,\ a_n=\frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

APPENDIX B:

Selected Properties of Laplace Transforms

Property Name	Property Relationship
Linearity	$\mathcal{L}\{a f(t)+b g(t)\}=a\mathcal{L}\{f(t)\}+b\mathcal{L}\{g(t)\}$
s-shifting	$\mathcal{L}\{e^{at} f(t)\}=F(s-a)$
Transform of Derivative	$\mathcal{L}\{f'\}=s\,\mathcal{L}(f)-f(0)$ $\mathcal{L}\{f''\}=s^2\,\mathcal{L}(f)-s f(0)-f'(0)$
Transform of Integral	$\mathcal{L}\{\int_0^t f(\tau)\,d\tau\}=\frac{1}{s}F(s)$
Differentiation of Transforms	$\mathcal{L}\{t f(t)\}=-F'(s)$
Integration of Transform	$\mathcal{L}\{\frac{f(t)}{t}\}=\int_s^{\infty} F(\tilde{s})\,d\tilde{s}$
t-shifting	$\mathcal{L}\{u(t-a)f(t-a)\}=e^{-as}F(s)$
Convolution	$\mathcal{L}^{-1}\{F(s)G(s)\}=f(t)*g(t)=\int_0^t f(\tau)\,g(t-\tau)d\tau$

Continued...

APPENDIX C:

Laplace Transforms for Selected t-domain Functions

$f(t)$	$F(s)=\mathcal{L}\{f(t)\}$
1	$1/s$
t	$1/s^2$
$t^n\ (n=1,2,3,\ldots)$	$n!/s^{n+1}$
e^{at}	$\dfrac{1}{s-a}$
$t^{n-1}e^{at}$	$\dfrac{(n-1)!}{(s-a)^n},\ n=1,2,\ldots$
$\cos at$	$\dfrac{s}{s^2+a^2}$
$\sin at$	$\dfrac{a}{s^2+a^2}$
$\cosh at$	$\dfrac{s}{s^2-a^2}$
$\sinh at$	$\dfrac{a}{s^2-a^2}$
$u(t-a)$	$\dfrac{e^{-as}}{s},\ a\geq 0$
$f(t-a)u(t-a)$	$e^{-as}\mathcal{L}(f)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$

Continued...

APPENDIX D:

Special Discrete Probability Distributions

Binomial Distribution, $X \sim b(x; n, p)$	
P.m.f.	$P(X = x) = {}^{n}C_{x} p^{x} q^{n-x}$, for $x = 0,1,\ldots,n$ and where $q = 1 - p$.
Mean	$E[X] = np$
Variance	$Var(X) = npq$
Hypergeometric Distribution, $X \sim h(x; N, n, k)$	
P.m.f.	$P(X = x) = \dfrac{{}^{k}C_{x} \times {}^{N-k}C_{n-x}}{{}^{N}C_{n}}$, for $x = 0,1,\ldots \min(n,k)$.
Mean	$E[X] = \dfrac{nk}{N}$
Variance	$Var(X) = \dfrac{nk}{N}\left(\dfrac{N-n}{N-1}\right)\left(1 - \dfrac{k}{N}\right)$
Poisson Distribution, $X \sim p(x; \lambda)$	
P.m.f.	$P(X = x) = \dfrac{\lambda^{x} e^{-\lambda}}{x!}$, for $x = 0,1,\ldots.$
Mean	$E[X] = \lambda$
Variance	$Var(X) = \lambda$

Continued…

APPENDIX E:

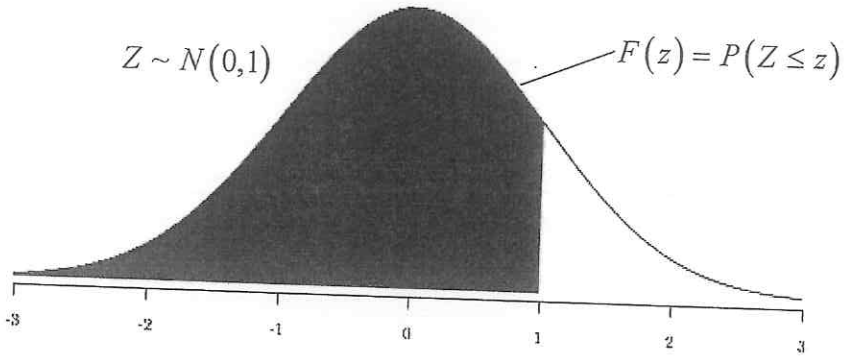
Special Continuous Probability Distributions

Continuous Uniform Distribution, $X \sim U(a,b)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \dfrac{a+b}{2}$
Variance	$Var(X) = \dfrac{(b-a)^2}{12}$
Exponential Distribution, $X \sim \exp(1/\beta)$	
P.d.f.	$f_X(x) = \begin{cases} \dfrac{1}{\beta} e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$
Mean	$E[X] = \beta$
Variance	$Var(X) = \beta^2$
Normal Distribution, $X \sim N(\mu,\sigma)$	
If X is any normal random variable where $X \sim N(\mu,\sigma)$, then the transformation $$Z = \frac{X-\mu}{\sigma}$$ yields a *standard* normal variable where $Z \sim N(0,1)$.	

Continued...

APPENDIX F:

Cumulative Standard Normal Distribution



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

End of paper.